DOES THE PPP NEED THE UIP?

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Introduction

The relative version of purchasing power parity is represented as:

$$E = k + P - P^* \qquad [1a]$$

where E is the logarithm of the exchange rate measured in units of currency A per unit of currency B, P is the logarithm of the price level in country A, P* is the logarithm of the price level in country B and k is a constant term.

This relationship has been empirically analized either imposing homogeneity restrictions on all the coefficients (unit root tests of the real exchange rate), on some of the coefficients (cointegration test between the two prices in common currencies) or without imposing any restriction on the coefficients. The conclusions obtained by the previous literature varies mainly as a function of the period analyzed and of the empirical methodology adopted (for a survey, Froot and Rogoff , 1995).

In this paper we focus on the post Bretton Woods period and analyze whether a relationship like (1a) is accepted by the data for Italy, United States and Germany. We adopt a multivariate system approach in which, initially, we test for cointegration and then we try to identify a cointegration space in which we have the PPP relationship (the "Johansen approach"). The studies that have adopted this approach have always rejected the PPP in favour of a long run relationship between the real exchange rate and the interest rate differential (for example, Juselius, 1995; Sjoo, 1995).

On the contrary, our conclusions are in favour of the PPP for all the cases considered when we allow for a structural break in the data. We arrive to this conclusion, after having identified the cointegration space in two different ways: one in which we have the PPP as a cointegrated vector and one in which the real exchange rate plus the interest rate differential is a cointegrated vector. Adopting a dominance criterion we choose the former identification.

The data and their univariate properties

The variables we use to test the PPP and its relationship with interest rates are quarterly sampled for the period 74:1, 92:4. We focus on the Italian, the German and the United States consumer price indexes (P_{ita}, P_{ger},P_{usa}) and the respective three-month treasury bills interest rates (R_{ita}, R_{ger},R_{usa}) ; the exchange rates are spot bilateral rates between Italian Liras and Dollars, German Marks and Dollars, Liras and Marks (E_{itusa}, E_{gerusa}, E_{itager})[1]. Prices and exchange rates are in logarithms, while the interest rates have been trasformed as: $I_j=\log(1+R_j)$.

A preliminary univariate analysis of the series, performed by the Augmented Dickey-Fuller (ADF) test (tab.1), emphasizes, as expected, the presence of one unit root in all variables. The null of a second unit root cannot be rejected at a significance level of 5% for all price series (ADF test on first differences). However, the presence of a second root is rejected if we adopt the SM (Schmidt and Phillips) test or the PP (Phillips and Perron) test. This ambiguity is common in the literature on unit root tests on price series[2]. We adopt the following solution to this ambiguity: the graphical inspection of all price series (Graph 1, 2, and 3) reveals the existence of a drift and of a break in 1982, determining a reduction in the slope of the drift, as a consequence of the beginning of a period of lower inflation in the industrialized economies, essentially due to the stabilization after the two oil price crisis.

This break might cause a misleading evidence of I(2)-ness in a I(1) series [Perron, 1989]. As a consequence we adopt Perron approach to perform a unit root test controlling for an exogeneously imposed structural break. The test is performed on the first difference of each price series adopting the additive outlier specification with a dummy variable of the tipe $D_t = \begin{cases} 0 & \text{if t} < 82{:}3 \\ 1 & \text{if t} \geq 82{:}3 \end{cases}$. The results (tab.1, AO-ADF) reject the I(2) hypothesis for all price series. As a consequence, in the following analysis we will introduce the deterministic variable D_t in the specification of the system.

Does the PPP hold in isolation ?

To analyze the existence of a long-run relationship among the variables in equation (1a), we adopt the Full Information Maximum Likelihood (FIML) cointegration approach developed by Johansen (1995). We analyze the validity of the PPP between Italy and USA, Germany and USA, Italy and Germany . In the light of the considerations of the previous section, we have constructed and estimated for each pair of countries a VAR(p) (in a reparametrized ECM form), including an unrestricted constant δ, that describes the presence of a drift in the series in levels, and a dummy variable D, that describes the 82.3 break in the price series:

$$\Delta \mathbf{Y_t} = \boldsymbol{\delta} + \psi K_{\mathbf{t}} + \boldsymbol{\gamma} \mathbf{D_t} + \Gamma_1 \Delta \mathbf{Y_{t-1}} + \Gamma_2 \Delta \mathbf{Y_{t-2}} + \ldots\ldots + \Gamma_{\mathbf{p}-1} \Delta \mathbf{Y_{t-p-1}} + \Pi \mathbf{Y_{t-1}} + \boldsymbol{\varepsilon}_{\mathbf{t}} \qquad [2]$$

where the terms ΔY_j , δ, and D are nx1 vectors, γ and Γ_j are nxn matrices, K is an (s-1)x1 vector of centred seasonal dummies, ε_t ~i.i.d. N(0,Ω) and , under the cointegration hypothesis, the nxn matrix Π can be factorized as $\Pi=\alpha\beta'$where α and β are nxr matrices of rank r.

The matrix β contains the r cointegrating vectors, while the matrix α contains the so-called loading factors that measure the speed of adjustment of each equation of the system to the different long-run relationships.

Since the critical values for the cointegration rank tests depend on the specification of the VAR and the standard tabulated values in Osterwald and Lenum (1992) are produced for a model including only a constant or a drift and not an intervention dummy among the deterministic term , we had to obtain the correct critical values by simulation[3].

Then, we performed the standard Johansen's analysis whose results are presented in table 2 : for each pair of countries we have included two lags in the VAR system [4].

The cointegration rank trace tests (conducted along the method of Pantula (1989) and Johansen (1995), starting from the hypothesis of rank=0 and increasing the assumed rank step by step), reveal the existence of one cointegration relationship between prices and exchange rates in the Italy/Usa and Italy/Germany cases, while in the Germany/Usa case the hypothesis of cointegration rank equal to zero, against the alternative of 3, cannot be rejected.

However, existence of cointegration is not a sufficient condition for PPP. From equation [1a] it is evident that also proportionality and simmetry restrictions should be satisfied. Hence, we perform an LR test to check the validity of these restrictions on the cointegrating vector. The results in table 3 show that the null hypothesis is strongly rejected. Therefore we can conclude that in the flexible exchange rate period, one cannot find any evidence supporting the standard PPP hypothesis for the cases considered in the simple VAR model that we have specified. On the other hand, in two of the three cases we have found that a "weak" version of the PPP holds (i.e. there is some long run equilibrium relationship between prices and echange rates).

In the light of this failure for PPP to hold as an isolated relationship, part of the recent empirical literature [Johansen and Juselius, 1992; Jore, Skjerpen and Swensen, 1992; Juselius, 1995; Sjoo, 1995; Caporale, Kalyvitis and Pittis, 1995], has tried a different approach by introducing into the VAR also interest rate variables. A theoretical basis for this approach comes from the "overshooting" models of exchange rate by Dornbush (1976) and Frankel (1979)[5].

In this new framework the long run relationship becomes:

$$(E - P + P^*) = \gamma (I - I^*) \qquad [1b]$$

Two of the hypothesis behind the derivation of this equation are the PPP and the uncovered interest parity (UIP).

The augmented system: allowing for interest rate differential

For each pair of countries we specify a VAR with five variables (P_i, P_j, E_{ij}, I_i, I_j), an unrestricted constant, the intervention dummy in 82.3 and centered seasonal dummies.

The cointegration rank trace tests (tab.4) show the existence (at a 5% significance level) of two cointegrating vectors between Germany and USA and three cointegrating vectors between Italy and Germany.

The same test seems to suggest that at 5% there exist four long run stationary relationships between Italy and USA, while at a significance level of 2.5% the evidence is in favour of a three dimensional cointegration space; our final choice of rank 3 is motivated by three different considerations.

From the theoretical point of view, there are only three meaningful equilibria in a five-dimensional VAR with prices, interest and exchange rates : a PPP relationship, the interest rate spread, and the Dornbush relationship combining PPP and UIP. Secondly, from the empirical point of view, the exam of the roots of the companion matrix of the system, after the imposition of rank three (tab 5), does not reveal any further unit root than the two that we have explicitely imposed; it's only possible to detect two complex roots (and not one root) with modulus 0.905 that is quite far from one. Finally, Reimers (1991) found that the Johansen cointegration test, in small system, and with small samples, over-rejects the null hypothesis even if it's true so that a less strict interpretation of the test could be advisable.

The estimated unrestricted β and α matrices in the three bilateral cases are presented in table 6.

The Johansen procedure produces an exact identification of the cointegrating space by means of a simple algebric procedure of normalization; however, the β-vectors spanning the cointegration space usually dont' have an economic interpretation. Johansen (1995) has provided some useful criteria to identify, both in formal, in empirical and in economic sense, the cointegration space .

In all the three bilateral cases, we have imposed a set of overidentifying restrictions on the sp(β) derived from some theoretical considerations and from the observation of the unrestricted estimated β, and we have verified if they respect the formal and empirical identification conditions.

Focusing on the Italy/USA case, the linear restrictions that we have built to identify the three cointegration vectors are defined in their explicit form by the following three matrices :

$$\mathbf{Q}_1^1 = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \qquad \mathbf{Q}_2^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{Q}_3^1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad [3]$$

The first matrix defines β_1 (i.e. the first cointegrating vector) as the Dornbusch equilibrium relationship [1b] combining PPP and UIP; the second one describes β_2 as the weak version of PPP that we have detected in the VAR without interest rates of the previous paragraph and the third matrix represents a cointegrating relationship between the italian interest rate, the USA interest rate and the exchange rate. The set of hypothesis in [3] formally (over)identifies the sp(β) (it satisfies the necessary and sufficient conditions for formal identification [Johansen, 1995] and is empirically accepted (empirical identification in Johansen, 1995) since the corresponding value of the Likelihood Ratio test statistic is $\chi^2(1) = 0.598$, with a P-value of 0.439 (tab.8).

The estimated restricted vectors and the corresponding loadings coefficients are presented in table 7 : we can remark that in general the interest rates and partially the exchange rates show higher speed of adjustment to the long run equilibria than the price variables, due to the larger stickiness of the latter.

We have tested an alternative set of overidentifying restrictions that does not involve any Dornbusch's relationship but implies only a PPP defined without allowing for any role of the interest rates in the long run. These restrictions are represented by the following matrices :

$$\mathbf{Q}_1^1 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{Q}_2^1 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{Q}_3^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad [4]$$

The first restricted vector is a PPP, the second describes a relation between P_{usa}, I_{ita} and I_{usa}, while P_{ita}, E_{itusa} and I_{ita} are the cointegrated variables in the third vector; the restricted estimates are reported in table 7.

The new set of linear restrictions (over)identifies formally and empirically the cointegration space (LR test = 3.6) (tab.8).

In summary, we have identified the cointegration space in two different ways. On one side, one of the identified cointegrating vectors is the Dornbusch relationship, combining PPP and UIP; on the other side, we find the PPP to be one of the cointegrating vectors.

To discriminate between the two competing identificated structures, we adopt the Likelihood Dominance Criterion of Pollak and Wales (1991). The idea is that, given two nonnested hypothesis H_1 and H_2 regarding the specification of the sp(β), we can select the dominant one by simply comparing their associated adjusted likelihood values.

By referring to a range of composite hypothesis H_c with different parametric sizes, the Dominance criterion acts as follow :

- H_1 is preferred to H_2 if L_2-L_1 < $[\chi^2(n_2+1)-\chi^2(n_1+1)]/2$
- H_2 is preferred to H_1 if L_2-L_1 > $[\chi^2(n_2-n_1+1)-\chi^2(1)]/2$
- The criterion is indecisive if $[\chi^2(n_2-n_1+1)-\chi^2(1)]/2 > L_2-L_1 > [\chi^2(n_2+1)-\chi^2(n_1+1)]/2$

where n_1 and n_2 are respectively the degrees of freedom related to the H_1 and H_2 and L_1 and L_2 are their respective likelihood values .

We refer to specification [3] as to H_1 and to specification [4] as to H_2 and using the likelihood values reported in table 8, we have: L_2-L_1 =1.5, $[\chi^2(n_2+1)-\chi^2(n_1+1)]/2 = [\chi^2(3)-\chi^2(2)]/2 = 0.91$ (at the 5% level) and $[\chi^2(n_2-n_1+1)-\chi^2(1)]/2 = [\chi^2(2)-\chi^2(1)]/2 = 1.075$.

Therefore, the Pollak and Wales criterion establishes the dominance of the 'overidentifying restrictions set' involving the PPP relationship.

Hence, in an augmented VAR we have been able to detect the PPP. It should be stressed that the role of the interest rates is still relevant even if they don't appear in this cointegrating vector, since they are allowed to influence the short run dynamics[6].

In the Germany/Usa case, a first attempt to (over)identify the cointegration space can be made by the imposition of the restrictions described by the following matrices and involving a Dornbusch relation and a relation between P_{ger}, E_{gerusa}, I_{ger}:

$$\mathbf{Q}_1^2 = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \qquad \mathbf{Q}_2^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad [5]$$

This set of restrictions satisfies the conditions for the formal identification but it generates a value of the Likelihood Ratio test statistic of 8.02, with a P-value equal to 0.046. A strict interpretation of this result should induce a rejection of such an identification.

Another overidentifying structure can be defined in the following way (estimates of the restricted vectors are in table 7):

$$\mathbf{Q}_1^2 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{Q}_2^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad [6]$$

It defines a PPP and a relation between P_{ger}, E_{gerusa}, I_{ger}, I_{usa}. This set of restrictions cannot be rejected by the Likelihood Ratio test for empirical identification (tab. 8).

Therefore we can unambiguously conclude that between Germany and USA the Dornbusch relation does not hold as a stationary relation, while the PPP is one of the equilibria described by the overidentified cointegration space.

Finally, we focus on the bilateral analysis between the two SME economies Italy and Germany. Again the starting identification structure that we impose on sp(β) includes the Dornbush relation and it can be fully described by the same set of matrices of the [3].

This structure is not rejected since the LR test is 2.58 (tab. 8). A different identification imposes the PPP to the first vector, a relationship between P_{ita}, E_{itager}, I_{ger} on the second one and a relationship between P_{ger}, I_{ita} and I_{ger} on the third one:

$$\mathbf{Q}_1^3 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{Q}_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{Q}_3^3 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad [7]$$

The necessary and sufficient conditions for the formal and empirical identification are satisfied by [7] (Tab.8).

Again we have two competing identified structures and to select one of the two we look at the Dominance Criterion : in this case (table 8), we have L_2-L_1 =1.45, $[\chi^2(n_2+1)$- $\chi^2(n_1+1)]/2$ = $[\chi^2(3)$- $\chi^2(2)]/2$ = 0.91 (at the 5% level) and $[\chi^2(n_2 - n_1 + 1)$- $\chi^2(1)]/2$ = $[\chi^2(2)$- $\chi^2(1)]/2$ = 1.075 and therefore the overidentification hypothesis based on the PPP is again preferred to the one based on the Dornbusch relation.

Conclusions

In this paper we have tested the hypothesis that the PPP holds as a long run stationary relationship. The analysis has been performed relatively to the three bilateral cases Italy/USA, Germany/USA and Italy/Germany and covers the post Bretton Woods period.

We have adopted a multivariate Full Information Maximum Likelihood approach to investigate the existence of a cointegrating relationship. In the analysis we have found some indication of I(2)-ness of the price series. Rather than adopting an I(2) system and, consequently, a multicointegration approach, we have interpreted the mixed evidence in favour of a second unit root as a consequence of the presence of one exogeneous structural break in the series corresponding to generalized reduction in inflation rates at the beginning of the 80s. The VAR model includes therefore a step dummy controlling for this break.

Differently from the recent literature adopting an approach similar to ours, we have been able to identify a valid cointegrating relationship that corresponds to PPP. This has been done within a system in which, from the point of view of the PPP, interest rate variables play a role only in the short run. Moreover, we also find an alternative identification structure in which, similarly to part of the recent literature, there is a cointegrating vector involving the real exchange rate and the interest rates differential. Applying a likelihood dominance criterion we found evidence in favour of the former identification.

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Notes

[1] All data have been obtained from Datastream

[2] For example, for the Italian case, Hamilton (1994) considers quarterly data for the period 73-89 as having only one unit root. On the other hand, Paruolo (1993) cannot reject the presence of two unit roots for the same variable on the period 70-91.

[3] We have utilized DisCo by Johansen and Nielsen (1993). The simulation was performed with 10.000 iterations and the number of the discretizations of the Brownian Motions, representing the asymptotic non standard theoretical distribution of the test, has been set at 600; the obtained critical values are reported in table 2 .

[4] The analysis has been performed in RATS with Malcolm (Mosconi, 1995) and with PcFIML (Hendry, 1994)

[5] For a formal derivation of the relationship between exchange rate, prices and interest rates see Helg-Serati (1996).

[6] The FIML Johansen estimates are obtained by a multi-step concentration of the likelihood function of the system with respect to different blocks of parameters and the long run coefficients estimates are function of the short run ones.

Tables

TAB. 1

Preliminary tests on the Integration order of the series

	ADF test	PP test	SP test	AO-ADF
P_{ita}	-2.74 (5)			
P_{usa}	-1.76 (5)			
P_{ger}	-0.63 (5)			
E_{itusa}	-2.24 (5)			
E_{gerusa}	-1.79 (5)			
E_{itager}	-2.58 (5)			
I_{ita}	-1.88 (5)			
I_{usa}	-1.71 (5)			
I_{ger}	-1.94 (5)			
ΔP_{ita}	-2.56 (5)	-3.4 (5)	-3.2 (5)	-4.52 (2)
ΔP_{usa}	-1.94 (5)	-3.27 (5)	-3.1 (5)	-5.00 (2)
ΔP_{ger}	-2.26 (5)	-5.46 (5)	-5.54 (5)	-6.72 (2)

Notes:
- the specification adopted in the tests is the one containing a costant .
- in brackets there is the number of lags included to remove autocorrelation in residuals

TAB. 2

Cointegration tests on the Price/Exchange rates models

	Rank	Constant	Trend	Statistic	Tabulated Value
	0	Unrestricted	Excluded	45.51	26.88
Italy/USA	1	Unrestricted	Excluded	9.18	9.54
	2	Unrestricted	Excluded	0.70	#### *
	0	Unrestricted	Excluded	14.93	26.88
Germany/USA	1	Unrestricted	Excluded	3.63	9.54
	2	Unrestricted	Excluded	0.12	#### *
	0	Unrestricted	Excluded	34.26	26.88
Italy/Germany	1	Unrestricted	Excluded	10.76	9.54
	2	Unrestricted	Excluded	0.35	#### *

Notes:
- all the VARs include two lags
- the tabulated values have been obtained by simulation with the Package DisCo.
* we treat the drift and the intervention dummy as unrestricted deterministic term and in this case, the drift term is not proportional to the value of convergence of the dummy function in the simulation interval [0,1] so that we cannot simulated the critical values for the hypothesis of rank =2 and a number of common trends less than two.

TAB. 3

Testing PPP hypothesis

	CHI SQUARE TEST	P-Value	Outcome
Italy/USA	23.32 (2)	0.0000086	rejected
Italy/Germany	6.81 (2)	0.033	rejected

Notes:
- we reject the null hypothesis if the P-Value exceeds 0.05
- in brackets the number of degrees of freedom.

TAB. 4

Cointegration rank tests

	Rank	Constant	Trend	Statistic	Tabulated Value 5%	Tabulated Value 2.5%
	0	unrestricted	excluded	114.09	70.44	73.96
	1	unrestricted	excluded	66.30	46.90	50.02
Italy/Usa (3)	2	unrestricted	excluded	29.46	26.88	29.68
	3	unrestricted	excluded	10.79	9.54	11.15
	4	unrestricted	excluded	0.41	####	####
	0	unrestricted	excluded	121.12	70.44	73.96
	1	unrestricted	excluded	47.05	46.90	50.02
Germany/USA (4)	2	unrestricted	excluded	21.10	26.88	29.68
	3	unrestricted	excluded	1.40	9.54	11.15
	4	unrestricted	excluded	0.01	####	####
	0	unrestricted	excluded	129.69	70.44	73.96
	1	unrestricted	excluded	71.42	46.90	50.02
Italy/Germany (4)	2	unrestricted	excluded	32.94	26.88	29.68
	3	unrestricted	excluded	7.08	9.54	11.15
	4	unrestricted	excluded	1.26	####	####

Notes:
- for the non available tabulated value relative to the hypothesis of Rank=4 see Tab.2
- in brackets there is the number of lags caracterizing the VAR system

TAB. 5

Roots (in modulus) of the Italy/USA model under Rank=3 hypothesis

1.00
1.00
$\pm$ 0.90
0.73
$\pm$ 0.68
$\pm$ 0.65
$\pm$ 0.58
$\pm$ 0.42
0.34
0.27

TAB. 6

Unrestricted estimates of α and β

	Estimated β_1	Estimated β_2	Estimated β_3	Estimated α_1	Estimated α_2	Estimated α_3
	1.00	1.00	1.00	0.01	-0.04	-0.04
	-2.34	-1.72	-1.55	-0.004	0.006	0.04
Italy/USA	-0.76	-0.13	-0.45	0.10	-0.74	0.35
	1.56	0.10	-0.19	-0.28	-1.71	-0.40
	0.39	0.05	0.15	-0.12	2.13	-0.12
	1.00	1.00		-0.06	0.01	
	-0.39	-0.64		-0.02	0.18	
Germany/USA	-0.04	-0.14		-0.03	1.01	
	-0.13	0.006		2.68	0.54	
	0.04	0.08		2.83	0.61	
	1.00	1.00	1.00	0.01	-0.02	-0.006
	-6.78	-2.17	13.31	0.01	0.008	0.0002
Italy/Germany	0.73	-0.25	-7.07	-0.06	-0.21	0.03
	0.40	-0.54	-0.84	-0.41	0.06	0.05
	0.57	-0.00002	0.04	-0.69	0.05	-0.08

TAB. 7

Estimates of the (over)identified cointegration vectors (normalized)

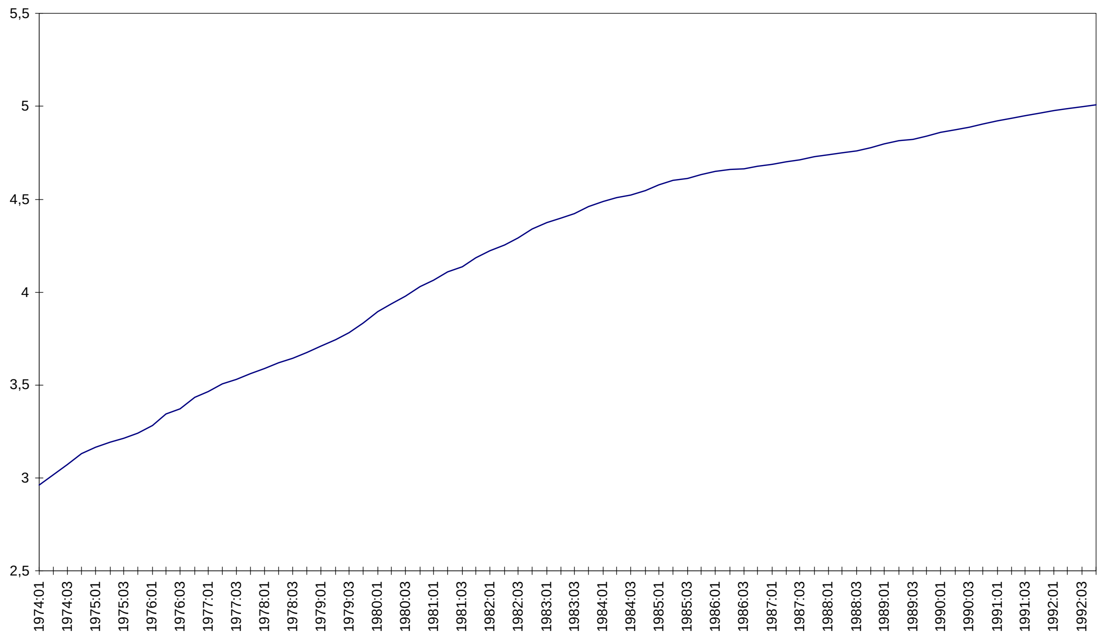
	Estimates under [3]	Estimates under [3]	Estimates under [3]	Estimates under [4]	Estimates under [4]	Estimates under [4]
	β_1	β_2	β_3	β_1	β_2	β_3
	1.00	1.00	0.00	1.00	0.00	1.00
	-1.00	-1.71	0.00	-1.00	1.00	0.00
Italy/USA	-1.00	-0.008	1.00	-1.00	0.00	-2.56
	-0.32	0.00	-0.80	0.00	-1.35	0.20
	0.32	0.00	-0.41	0.00	-0.35	0.00
	Estimates under [5] β_1	**Estimates under [5]** β_2	**Estimates under [5]** β_3	**Estimates under [6]** β_1	**Estimates under [6]** β_2	**Estimates under [6]** β_3
				1.00	1.00	
				-1.00	0.00	
Germany/USA				-1.00	0.57	
				0.00	-0.22	
				0.00	0.07	
	Estimates under [1] β_1	**Estimates under [1]** β_2	**Estimates under [1]** β_3	**Estimates under [7]** β_1	**Estimates under [7]** β_2	**Estimates under [7]** β_3
	1.00	1.00	0.00	1.00	1.00	0.00
	-1.00	8.55	0.00	-1.00	0.00	1.00
Italy/Germany	-1.00	-4.92	1.00	-1.00	-1.36	0.00
	-0.13	0.00	-1.00	0.00	0.00	-0.44
	0.13	0.00	-0.54	0.00	-0.07	-0.22

TAB. 8

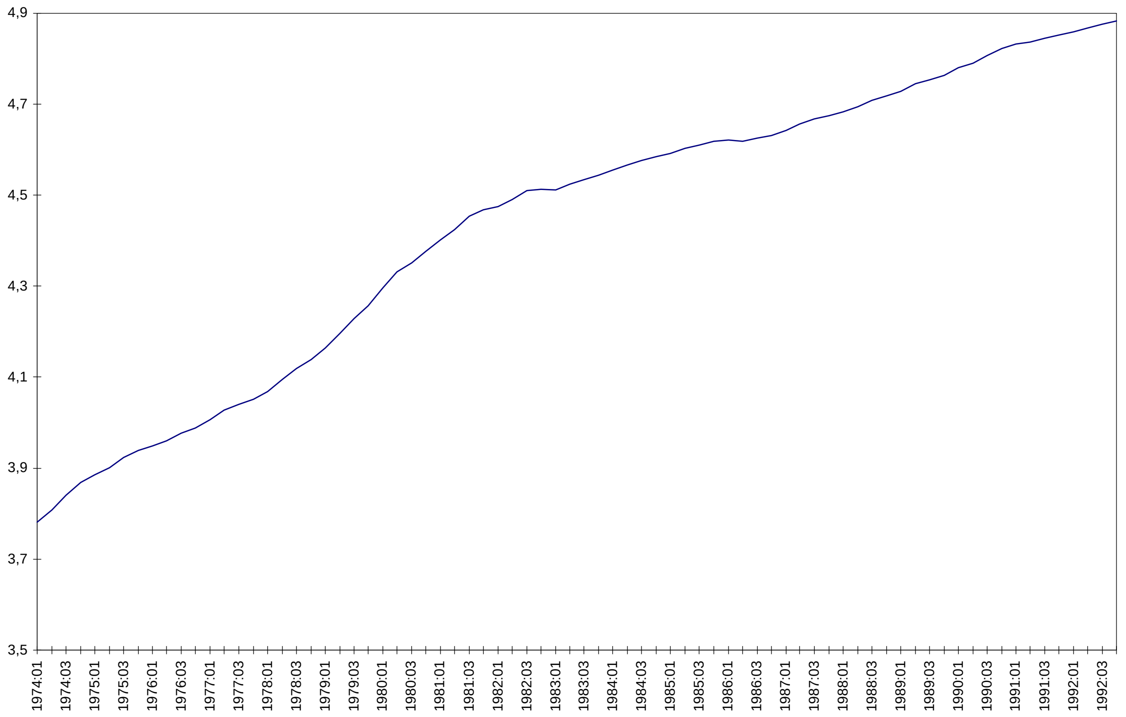
Test on the empirical identification of sp(β) : Likelihood Ratio tests

Null Hypothesis	Likelihood of the restricted model	Degrees of freedom	Test statistic	P-Value	Outcome
[3]	1386.39	1	χ^2=0.598	0.439	non rejected
[4]	1384.89	2	χ^2=3.60	0.16	non rejected
[5]	1458.85	3	χ^2=8.02	0.046	rejected
[6]	1459.13	3	χ^2=7.45	0.059	non rejected
[3]	1486.49	1	χ^2=2.58	0.108	non rejected
[7]	1485.04	2	χ^2=5.49	0.064	non rejected

Graph.1: Italian consumer price index



Graph.2 : United States consumer price index



Graph.3 : German consumer price index